

Di Ning

**Quiz 1**  
**VECTOR CALCULUS**  
**MATH 21D, Sect 002, Winter Quarter, 2013**  
**INSTRUCTOR: Blake Temple**

1. Find the volume of the solid in the first octant bounded by the coordinate planes, the plane  $x=3$ , and the parabolic cylinder  $z=4-y^2$ . (10 points)
2. Sketch the region bounded by the given lines and curves. Then express the region's area as an iterated double integral and evaluate the integral. (10 points)

The parabola  $x=-y^2$  and the line  $y=x+2$

1. Solution:

first octant,  $x \geq 0, y \geq 0, z \geq 0$

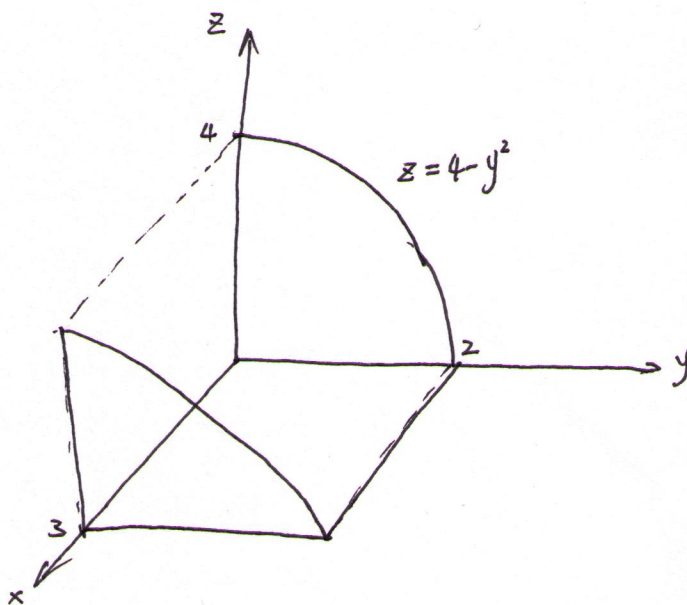
$$0 \leq x \leq 3$$

$$0 \leq y \leq 2$$

$$V = \int_0^3 \int_0^2 (4-y^2) dy dx$$

$$= \int_0^3 (4y - \frac{1}{3}y^3) \Big|_0^2 dx$$

$$= 3 \times (8 - \frac{8}{3}) = 16 \quad \text{= (5pts)}$$



2. Solution:

$$x = y - 2$$

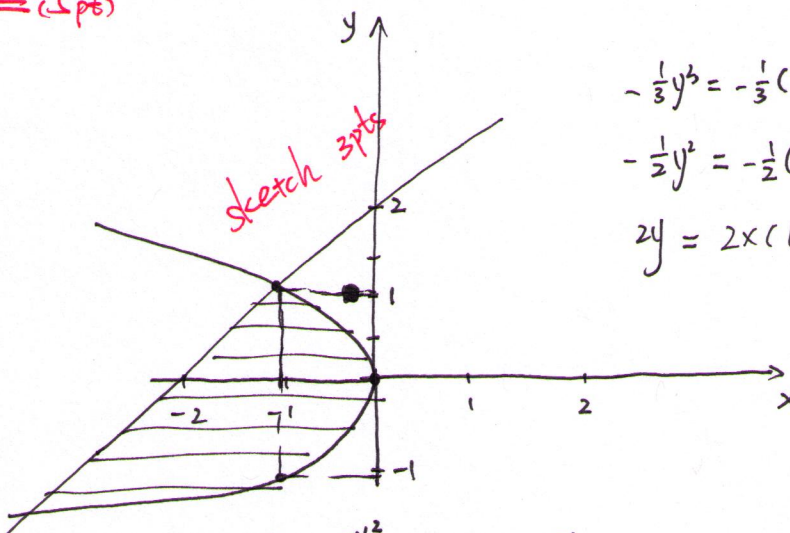
$$x = -y^2$$

When  $y - 2 = -y^2$ .

$$y^2 + y - 2 = 0$$

$$(y+2)(y-1) = 0$$

$$y_1 = -2 \text{ and } y_2 = 1$$



$$-\frac{1}{3}y^3 = -\frac{1}{3}(1+8) = -3$$

$$-\frac{1}{2}y^2 = -\frac{1}{2}(1-4) = \frac{3}{2}$$

$$2y = 2 \times (1+2) = 6$$

$$\int_{-2}^1 \int_{-y^2}^{-y^2} dx dy = \int_{-2}^1 (-y^2 - y + 2) dy$$

$$= (-\frac{1}{3}y^3 - \frac{1}{2}y^2 + 2y) \Big|_{-2}^1 = \frac{9}{2} \quad \text{= (5pts)}$$